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## ABSTRACT

A new basis for the quantitative study of ground combat is introduced that argues the inadequacy of attrition models and the need to incorporate the effects of suppression of the enemy with firepower. A quantitative approach to suppression of enemy fire is offered. Then an analysis shows that the effect of own fire in suppressing enemy fire will, in suitable, frequent circumstances, reverse the conclusions derived from the Lanchester square law, so that the squared term determining the victor is unit firepower instead of the numbers of units engaged.

A fundamental aim of physical science is to describe its processes with dynamic models, mathematical if possible. The same aim, often implicit, is true of descriptions of combat phenomena in military operations research. Models of military operations are necessarily more abstract and approximate than those of physical science. This is especially so because the scientific study of combat operations is complicated by human presence. The problem, of course, is to reduce the enormous effects of "human factors," to an understandable construct, or paradigm. An excellent statement of the problem is in Davis and Blumenthal's monograph, *The Base-of-Sand Problem: A White Paper on the State of Military Combat Modeling* [1991].

For more than a decade The Military Conflict Institute (TMCI) has been addressing the problem via a theory of combat that derives from six axioms. A goal of TMCI is to express the theory in a study, thus far unpublished.[1] As something of a status report, I undertook a Naval Postgraduate School research paper to digest what seemed to be the essence of TMCI's work. It is entitled "Combat Science: An Organizing Study" [1993]. The present paper is an exposition of a small but important consequence of the two endeavors.

We may come at our subject with the following question which arises from the remarkable results of the Gulf War:

> What analytical proposition helps to explain the evidence that much superior combat power when properly applied will result in disproportionately small losses to the winner while he achieves his objective?

The relevant principles from Combat Science are:

- Military force, or combat power, is a real phenomenon, the results of which are observed by its effects on the enemy in battle.
- The observable effects of combat power are not merely physical (producing casualties) but also mental (persuading the enemy of our superiority) and spiritual (diminishing enemy morale and will to continue fighting.)

For purposes of this paper, we will look carefully only at the most measurable manifestation of combat power's mental effect, which is suppression of enemy actions, specifically his return of fire. Its spiritual effect to diminish enemy morale plays no part in the computations that follow, but may be seen to be an unquantified bonus.

Let us begin with a fresh look at the Lanchester square law from the perspective of combat science. As it was conceived and employed by Chase [1902], Fiske [1905], Osipov [1915] and Lanchester [1915] the square law describes combat as a purely physical phenomenon:

[Unit fire's physical effect in casualties imposed/minute]

$x$

[# of physical units firing]

=

[Fewer enemy units]

Starting with Osipov, who reached his own unique conclusion as to the appropriate relationships, there have been many objections that the classical square law does not fit the historical battle casualty data for ground combat. The most commonly cited reason is (properly enough) that the law cannot apply when the required conditions are not present in the battle. The conditions are that each participant must be able to fire at each participant on the opposing side; and incapacitated opponents must be known at once, so that fire is distributed only against active opponents. Patently these conditions are seldom fully met, with not-so-obvious effects on the square law's applicability.

There are probably three additional major reasons, each related to the other two.

One is insufficient attention to defender advantage. Unit fire effectiveness will normally be greater on the defender side until the attack is nearly consumated. This advantage comes from the defender's superior posture. When identical unit effectiveness is (wrongly) assumed for attacker and defender the results will appear to show a less-than-square-law effect for the attacker's numerical advantage.

The second reason is that a battle is usually episodic, such that different elements of a force play predominant roles at different phases of the battle. A battle might involve disproportionate losses upon the attacker during his assault and disproportionate losses upon the defender during his attempt to disengage. Different weapons produce casualties more and less effectively during different phases. Historical battle data rarely specify casualty production in this way, and so the episodic effects are disregarded out of necessity.

The third reason is shortage of data with which to measure the effect of suppression of enemy actions from firepower.[2] Since suppression produces no casualties and is a transitory phenomenon that disappears when the battle is over, the effect of its presence is overlooked (or is merely implicit) in almost all combat models, including high resolution simulations. In Chapter 18 of *Understanding War,* T. Dupuy began his fine discussion of suppression with: "There is probably no obscurity of combat requiring clarification and understanding more urgently than that of suppression" [1987].

The heart of the problem is not a refusal to acknowledge the importance of suppression but the lack of an analytical approach that describes the phenomenon and its importance. We wish here to develop a quantifiable model of it. Then we will fulfill the purpose of this paper, which is to show how an appreciation of the cognitive influence of fire can reverse present conceptions of the value of numbers relative to the unit firepower of a fighting force.

We begin with the common form of the Lanchester square law:

$$\frac{dA}{dt} = -\beta B(t) \qquad \frac{dB}{dt} = -\alpha A(t) \qquad (1)$$

where $\alpha$ and $\beta$ are the *constant* unit effectiveness coefficients for A and B respectively, measured in kills per shooter per unit of time. A(0) and B(0) are the initial force strength; A(t) and B(t) are the forces remaining at any later time t; and dA/dt and dB/dt are the rate of losses of A and B respectively at time t.

A solution to equations [1] for end time T is the state equation:

$$\alpha\,[A(0)^2 - A(T)^2] = \beta\,[B(0)^2 - B(T)^2] \qquad (2)$$

Equation [2] shows the well known square law phenomenon, in which A will win a fight when the weaker side is annihilated:

$$\alpha\,A(0)^2 > \beta\,B(0)^2$$

and vice-versa. The classic conclusion from the square law is that the "fighting strength" (Lanchester's term) of a force increases in direct proportion to unit effectiveness and in proportion to the square of the number of forces engaged.

Notably, the casualty-producing effect of fire is regarded as a constant. There is ample evidence that unit effectiveness, $\alpha$ or $\beta$, is not even approximately constant but is highly variable. This is true when one is using casualty data taken battle after battle or day after day during a campaign: see for example, D. Hartley [1990, pp. 3-4; and 1991, p. 3]. It is true when one is using casualty data taken from different places on a battlefield: see G. Kuhn [1989], for example. It is true during different phases of a battle: see R. Helmbold [1979], for example.[3] Even though data by phase is seldom available, the variability of fire effectiveness is readily appreciated by reflecting upon a battle's sequence: for example, an artillery preparation phase, an armored assault phase, and an exploitation phase after an enemy position is overrun.

If $\alpha$ (t) and $\beta$ (t) are also variables, then the Lanchester square law equations must be written:

$$\frac{dA}{dt} = -\beta(t)\,B(t) \qquad \frac{dB}{dt} = -\alpha(t)\,A(t) \qquad (3)$$

Two important reasons that $\alpha$ (t) and $\beta$ (t) vary are: first, a change in range between the fighting units as the battle unfolds; and second, the suppressive effect of each side's fire on the other's fire. For two forces fixed in place with no change in range, the suppression suffered by A will be in consequence of the volume and accuracy of fire by B, and vice versa. Let us postulate a constant of proportionality, g, such that $g\beta(t)B(t)$ is the time-rate at which each A-shooter's fire is curtailed (measured in volume or accuracy or both). Similarly define a value, h, such that $h\alpha(t)A(t)$ is the rate that each B-shooter's effectiveness is diminished by A's fire.[4] We write:

$$\frac{d\alpha}{dt} = -g\beta(t)B(t) \qquad \frac{d\beta}{dt} = -h\alpha(t)A(t) \qquad (4)$$

The total effect of firepower represented by equations [3] and [4] is twofold. Fire will reduce both the numbers of enemy forces remaining and the lethal effectiveness of the remaining forces. Old equations [3] and new equations [4], however, have dimensions incompatible with each other. After reconciling the dimensions (to killing power lost per unit of time), it is possible to retain all quantities and proceed to explore the relationships between the four equations. There remains, of course, the serious problem of finding data, even rough data, for inputs.

## DEVELOPMENT

Let us proceed, now, to develop our principal point. As remarked above, casualties are a permanent consequence of fire, but suppression's effect is only transitory. We usually say combat forces are "pinned down" under fire, but not permanently affected. Arguably the changes in $\alpha$ and $\beta$ will be much greater during a battle than the changes in force size caused by attrition. In a typical land battle, casualties will be less than 10%.[5] See among many examples, R. McQuie, "Battle Outcomes: Casualty Rates as a Measure of Defeat" [1987].

As McQuie says, something other than casualties must cause the losing side to break off action in a typical battle. In many, perhaps most, battles mission success is obtained by the cumulative cognitive effects of fire on the enemy. The losing side's forces curtail their maneuvers and fire to avoid destruction. The losing side is also increasingly dispirited, with further loss of fighting effectiveness. Or the losing commander thinks that the situation can only go from bad to worse and withdraws to fight afresh from what he hopes will be a more favorable posture.

We may easily explore circumstances in which the volume and precision of fire have a much greater effect to suppress enemy fire than to impose casualties. Define b(t) as the rate of fire in shots by each unit B, and a(t) as the rate of fire by each A. Define s as the rate of reduction of fire imposed on the other side per shot fired, such that:

$$\frac{da}{dt} = -s\,B\,b(t) \qquad \frac{db}{dt} = -s\,A\,a(t) \qquad (5)$$

With equations [5] we measure combat power wholly by its effect on the volume and accuracy of the enemy's return fire during the course of the battle. The model, in its stylized purity, says that a battle is won by the side that sustains its own fire and suppresses that of the enemy. Casualties are an unmeasured by-product.

The state equation for [5] is entirely analogous to equation [2]:

$$A[a(0)^2 - a(T)^2] = B[b(0)^2 - b(T)^2] \qquad (6)$$

Equation [6] leads to a radically new "law" of combat in which the quantity of fire (measured by its suppressive effect) is the squared term. Now side A will win a battle in which the enemy's capacity to fire back is "annihilated" (fully suppressed) if:

$$A\,a(0)^2 > B\,b(0)^2$$

When the cognitive effects (on mind and will to win) predominate over casualties inflicted because the accuracy and intensity of fire cause the enemy's accuracy and intensity to deteriorate, under those circumstances effective unit firepower is more valuable than numbers engaged. The losing side will discontinue the battle because it is being *dominated* by the enemy's fire, will over time be reduced to impotence, and will eventually be destroyed if it does not concede the enemy's military objective. Under such circumstances the traditional Lanchester square law conclusion favoring numbers over quality is reversed, and unit firepower is more influential than numbers of units.

## A NUMERICAL EXAMPLE

As a numerical example, let A(0) = 10 units of force, and B(0) = 5 units. Let the initial destruction rate of A be $\alpha$ (0) = 1 kill of B per A unit per hour; and of B, $\beta$ (0) = 2 kills of A per B unit per hour. The two sides are equal at the outset in the sense that

$$\alpha A = \beta B = 10 \textit{ kills per hour.}$$

If the Lanchester square law applies, unit fire is constant and continuous, with $\alpha$ = 1 and $\beta$ = 2, and equation [2] obtains.

- A will win a fight to the finish, and when the B force is destroyed, A(T) = 7.07 units, so 71% of the A force survives. Such is the power of numbers under Lanchester square law conditions.
- If B senses defeat after one of its 5 units (20%) is lost and successfully ends the battle, A(T) = 9.05 when it is over. A has lost nearly one unit to B's superior fire, which is almost as much as B lost. A's long term cumulative advantage of numbers has not had time to take effect.

Next assume that the effect of suppression on enemy fire works much faster and is far more important than attrition. Equation [6] applies, so that in a formal sense neither side loses units. A = 10 and B = 5 are constants, and only the volume and accuracy of each side's fire is affected by the other's. Let the initial firing rate of A be a(0) = 10 rounds per minute per shoooter and let each round suppress enemy fire at s = .01 rounds per minute per enemy unit. Correspondingly, let b(0) = 20 rounds per minute with the same value of s. The two sides' initial volumes of fire are equal in that Aa(0) = Bb(0) = 100 rounds per minute.

- At time T when B has totally suppressed A's fire, each unit of B has an unsuppressed rate of fire remaining of b(T) = 1.414 rounds per minute, or 71% of the initial rate. We may presume A to be impotent and at the mercy of B.
- If A senses that B's volume of fire is dominant after his fire has been curtailed by 20%, he may end the battle and withdraw as he is able. At the time the battle is over, a(T) = 8 rounds per minute per shooter, and b(T) = 18.1 at its end, which is 90.5% of B's initial firepower.

## INTERPRETATION OF THE SUPPRESSION MODEL

An interpretation of the suppression computation is interesting in several regards.

First, by supposition B inflicts no casualties, and so all of A may withdraw and live to fight another day. It is possible that A's force is permanently disorganized and demoralized, but that is not inherent in the model.

Second, the battle is won by superior unit firepower, even though at its outset the total rate of fire and fire effectiveness of both sides were equal; unit effectiveness is more influential than the number of units.

Third, B wins by forcing A to concede B's tactical objective or else suffer complete suppression and, by presumption, destruction. Mission accomplishment is a good way to decide who won the battle, but it is not the usual way in analysis, which is to compare casualties. Probably B's organization and morale will be stronger after the battle than before it, but again, the model does not tell us.

Fourth, we should not conclude that A's situation is hopeless in future battles. Now that we know the significance of treating a and b (or $\alpha$ and $\beta$) as time-dependent variables, we may anticipate battlefield conditions in which A finds a stronger position with an improved firepower ratio and so is able to exploit its numerical advantage.

Fifth, observe that the victory went to the side whose fire *dominated* on the battlefield. In a formal sense this was true by postulation, for we assumed that attrition played no part. Nevertheless it is useful to look at how and why the lethal potential of B's superior fire was decisive. It was in part because B's fire attenuated A's fire more rapidly, and in part because after A is reduced to impotence he must surrender or face destruction. In war the activation and effective employment of superior firepower is the central cause of victory, whether or not casualties determine the outcome. (Our model ignores movement for the sake of analytical simplicity, but it is safe to say battlefield movement unsupported by covering, suppressive fire is a rare occurrence. Tactical maneuvering is achieved by an astute blend of covering fire and movement.)

Sixth, these models are formalisms, whether basic equations [1], [3] or [5] are used. For one thing, casualties will occur on both sides. For another, the reduction of fire on the losing side caused by the winner's fire will usually not go all the way to zero. There is a point of diminishing returns in the suppression effect of fire towards attenuating enemy fire.

Seventh, as a reminder, ground combat is our subject. Naval and air combat take their own form. In the judgment of the author, attrition is the essential phenomenon, and suppression as a driving cause of sea and air battle outcomes is rare.

## SUMMARY AND CONCLUSIONS

Let us summarize the major points of the quantitative analysis:

- The coefficients of unit effectiveness ought to be regarded as variables in time.
- Fire volume and unit effectiveness will often be far more variable than the change due to casualties, in which case they have a greater immediate effect on the outcome than attrition.
- When unit firepower is diminished more rapidly than the surviving number of units (due to the effectiveness of enemy fire) then the unit firepower advantage has the square law effect, not numerical advantage.

It is quite reasonable to expect that the suppression effect of superior combat power will frequently be greater than the attrition effect. Referring again to McQuie, at the end of 80 battles in and after World War II, the median casualties were 4% at the time the attacker abandoned his objective and 8% when the defender conceded defeat. McQuie cites the decisive effect of maneuver (or by implication the inability to maneuver) as the salient reason for termination by the loser in the majority of the battles. We have not examined the suppression of maneuver, though it is possible to do so. In any case, McQuie's data suggest that casualties are seldom the determinant of battle outcome. They also suggest that when the attacker's success results in a rout or surrender of the enemy, casualties may be principally in the form of prisoners taken instead of bodies slain.

One should not go so far as to disregard attrition. But even if one regards casualties as the available and hence utilitarian measure, nevertheless the Lanchester square law does not follow with its conclusion that numerical advantage is more valuable than unit firepower. Indeed, in recent research to demonstrate the utility of his "defender's advantage parameter," R. Helmbold [1995] shows that numerical advantage, which is the most important determinant in the square law, is among the worst predictors of victory in 83 historical battles. One should regard attrition and suppression both as important. This analysis suggests, to say the least, that their source—firepower—is more influential that numbers engaged.

But the answer we offer here to the question posed in the beginning is that sharply superior combat power must impose and exploit suppression when the goal is a relatively bloodless victory. The best example in the past 50 years of victory by suppression is the German blitzkrieg of 1940 in France. The blitzkrieg had many of the properties of the Gulf War. No attrition model can explain the blitzkrieg phenomenon, which was achieved by an intensive, local suppression of the defenders' ability to resist during a German armored-mechanized breakthrough. The breakthrough was followed by extensive demoralization of the French defenders when they faced an enemy in their rear. Blitzkrieg (a.k.a. lightning war or maneuver warfare) worked on the minds and spirits of the enemy to reduce his combat power to near zero.

It is in the nature of a successful campaign of maneuver that few battles are fought and the casualties that attract the interest of military historians and analysts are largely absent. To understand the full value of very superior combat power one must look for Sherlock Holmes' dog that didn't bark: mission attainment with the near-absence of combat and casualties. Perusal of battles analyzed quantitatively reveal few if any for the entire 1940 German campaign in France. For example, *Combat History Analysis Study Effort: CHASE*, by R. Helmbold [1986] lists only one, Sedan-Meuse River of 13 May 1940. Another example is the Japanese defeat of the British on the Malay Peninsula in 1941 in a brilliant campaign of maneuver with never a battle worthy of the history books. These are examples of what Sun Tzu told us long ago is the true measure of successful generalship [1963, p. 77]. We recall what Clausewitz said in the same vein: it is not the attacker but the defender who initiates a battle; the defender may choose to concede the opponent his aim instead of fighting [1976, p. 377].

One may well ask why American military tacticians should not long ago have perceived the quite reasonable, if not obvious, relationships described above. It is probably a proclivity to leap at once to the "practical:" an ad hoc perspective and disinterest in general theory. Since 1950 the practical problem under study has been "how to fight outnumbered and win," tied to the NATO central front in Europe. The advantage of superior firepower, its potential for reducing casualties, and the deeper effects of the phenomena of combat power were not ours to exploit until the Gulf War.

The best military leaders have always sought victory by domination and control, not bloodshed. We analysts do a disservice if our studies cannot carry the quantitative analysis of battle outcomes beyond the measurement of casualties. The U. S. government seems to be in the process of casting aside the means to apply overwhelming combat power in the next major regional conflict. It would be most timely to give some attention to suppression as a root cause and dominance as the vital effect in determining the highly favorable outcome of the ground campaign in the Gulf War, and many others.

## FUTURE RESEARCH

The Army and Marine Corps espouse "maneuver warfare." The thoughtful reader will see the possibility of extending the model herein in a number of ways that lead to a better understanding of maneuver's advantage, and the relationship between fire and movement.

a. The first and simplest is to work out the mathematics when both attrition and suppression result from fire; in other words, when fire reduces enemy numbers and return fire simultaneously. Some parametric analysis of the relationships will then be possible, but eventually the more difficult task of obtaining numerical values will have to be confronted. Clues to maneuver warfare are involved because one effect of suppressive fire is to "pin down" the enemy with one element of force (notably artillery and close air support), while another achieves a penetration (notably armor), or maneuvers on a flank (notably mechanized or other mobile forces).

b. The Lanchester square law form has been the basis of this discourse. There is no reason to think the square law is "right," except that aimed fire is the image of the hypothetical engagements described. I have been somewhat vague about the extent to which suppressive fire is aimed or fire-hosed. What we do know from Osipov, Helmbold, Hartley and others is that the square law advantage of numbers has rarely been achieved when measured by attrition. I think operations analysts of ground combat should wish to investigate whether historical results comport with combat models better after suppression is introduced. I do not know whether to suggest starting with the square law, the linear law, or something in between. Their instincts will be better than mine.

c. For the purpose of modeling maneuver warfare, a more direct approach is to extend the equations to include the suppression of movement as well as fire. On one hand, this is appealing because we know that fire inhibits enemy movement, yet combat models do not reflect it as a function of the accuracy and volume of enemy fire. On the other hand, one discovers a dimensionality problem and must wrestle with inputs in terms of the deceleration rate, which has units of meters/minute-enemy shot or the equivalent. If we analysts are serious about helping to explore the operations and tactics of maneuver warfare, it seems incumbent on us to try, by showing the connection between movement and supporting (suppressing) fire in maneuver warfare. For what it is worth, it is certainly possible to write equations in which fire has all three effects: casualties, suppression of fire, and suppression of movement.

d. Another approach would be to regard the effect of opening fire as a step function. In such a model, the instant fire is opened, enemy return fire (or motion) is diminished to a new, lower level. This is more or less implicit now in the inputs for the coefficients of unit effectiveness. The new feature would be to scale the effectiveness of B's fire (or motion) inversely with the accuracy and intensity of A's fire. To my knowledge, this has never been attempted. The conceptualization is not a big step, but the scaling factors would come only with great difficulty.

## ENDNOTES

1. To TMCI it seemed desirable to call their cornerstones Axioms, whether traced from an innate comprehension of truth or derived from common experience. The six axioms are thought to be the fewest possible, so obvious as to be indisputable, and so essential that no structure of theory or model should proceed without cognizance of them. Root definitions must, of course, accompany them:

Definition 1. A MILITARY FORCE is a set of elements which are activated for combat.

**AXIOM 1. COMBAT occurs by deadly interactions between military forces.**

Definition 2. COMMAND is the function which organizes, motivates, makes decisions regarding, and directs the activities of its force.

**AXIOM 2. In combat each COMMAND seeks to achieve a goal, called its mission, which has perceived value.**

Definition 3. COMBAT POTENTIAL is the latent capacity of a military force to achieve results in combat.

**AXIOM 3. COMBAT POTENTIAL is embodied in military forces.**

Definition 4. COMBAT POWER is the realized capability of a military force at an instant of time to achieve results in combat.

**AXIOM 4. Command activates its combat potential to create COMBAT POWER in furtherance of a mission.**

Definition 5. DOMINATION is the condition wherein one military force imposes its will on the other.

**AXIOM 5. In combat, DOMINATION of the opposing military force is the ultimate means of achieving an objective.**

**AXIOM 6. UNCERTAINTY is inherent in combat.**

The status of TMCI theory of combat may be had from the author at the Naval Postgraduate School, Monterey, California 93943, or Dr. Donald S. Marshall, Executive Director of The Military Conflict Institute, 12 Fairfield Street, Salem, Massachusetts 01970.

2. A source of some rare exceptions is MORS' two-volume Proceedings of the Second Mini-Symposium on the topic, *Human Behavior and Performance as Essential Ingredients in Realistic Modeling of Combat—MORIMOC II.* Among the noteworthy papers are:

a. David Rowland, "Assessment of Combat Performance with Small Arms"

b. Charles L. Frame, Brian R. McEnany and Kurt A. Kadivko, "Combat Operational Data Analysis: An Examination World War II Suppression Data"

c. George Schecter, James C. Richards and Henry A. Romberg, "Tactical Deterrent Effects Model"

d. Trevor N. Dupuy, "The Fundamental Information Base For Modeling Human Behavior In Combat"

3. On page 2 of this early paper by one of our most prolific and perceptive analysts of historical ground combat, Helmbold writes: "[Daniel] Willard's approach is typical of those which attempt to infer the form of Lanchester's equations from an analysis of a large number of battles for which the initial and final strengths on both sides are known, without depending on any information about the details of their attrition histories. A finding that this approach is fully justified would be of capital importance for the development of a theory of combat, because data on initial and final strengths are available for hundreds of battles. . .they at least are much

more readily available than data on attrition histories" [1979]. In the preceding paragraph Helmbold has just finished taking note of the famous effort by J. Engel to validate the Lanchester square law's *shape* as it unfolded during the extended Battle of Iwo Jima, so he is clearly concerned with the attrition *history* occuring within a battle, or short campaign.

4. A more thorough description would backtrack to equations [3] to explain that functions $\alpha$ and $\beta$ are really composites of a firing rate term in shots fired/minute, and an accuracy term, in hits per shot. Then in equations [4] we would have same firing rate value as in equations [3] plus a suppression term, in shots surpressed per shot fired.

5. In memorable and decisive battles, casualties have been as much as 25-35%, but such battles are usually episodic, comprising separate phases in which large changes of $\alpha$ and $\beta$ in effect *define* the phases.

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